Finite dimensional vector space

Vector Space

→ V the Given vector space.
→ u, v, w vectors in V
→ K the given number field.
→ a, b, c or k scalars in K

⇒ The Real line R has "dimension" one.
⇒ The cartesian plane R^2 has "dimension" two.
⇒ The space R^3 has "dimension" three.

Def – Let V be a non-empty set with two operations

(i) Vector Addition – $u, v \in V$ a sum $u+v$ in V

(ii) Scalar Multiplication $u \in V$, $k \in K$ a product $ku \in V$. Then V is called a vector space (over the field K) if the following axioms hold for any vectors $u, v, w \in V$.

Date
Page

(A_1) $(u+v)+w = u+(v+w)$

(A_2) There is a vector in V, denoted by 0 and called the zero vector, such that for any $u \in V$

$$u+0 = 0+u = u$$

(A_3) For each $u \in V$, there is a vector in V, denoted by $-u$ and called the negative of u such that

$$u+(-u) = (-u)+u = 0$$

(A_4) $u+v = v+u$

(M_1) $K(u+v) = Ku+Kv$ for any scalar $k \in K$

(M_2) $(a+b)u = au+bu$, for any scalars $a, b \in K$.

(M_3) $(ab)u = a(bu)$ for any scalar $a, b \in K$

(M_4) $1u = u$ for the unit scalar $1 \in K$.

Th 1. Let V be a vector space over a field K.

(i) For any scalar $k \in K$ and $0 \in V$, $k0 = 0$.

(ii) For $0 \in K$ and any vector $u \in V$, $0u = 0$

(iii) If $ku = 0$, where $k \in K$ and $u \in V$ then $k = 0$ or $u = 0$

(iv) For any $k \in K$ and any $u \in V$,

$(-k)u = k(-u) = -ku$.